Bohm's Quantum Potential

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The **Bohm** or **Bohmian interpretation** of quantum mechanics, which Bohm called the **causal**, or later, the **ontological interpretation**, is an **interpretation** postulated by **David Bohm** in 1952 as an alternative to the standard **Copenhagen interpretation**. The Bohm interpretation grew out of the search for an alternative model based on the assumption of **hidden variables**. Its basic formalism corresponds in the main to **Louis de Broglie**'s **pilot-wave theory** of 1927. Consequently it is sometimes called the **de Broglie-Bohm theory**.

The Bohm interpretation is **causal** but not **local** and is **non-relativistic**. It has not been disproven, but there are other schemes (such as the **Copenhagen interpretation**) that give the same theoretical predictions, so are equally confirmed by the experimental results.

The Bohm interpretation is based on these principles:

- **Every particle travels in a definite path**
- **We do not know what that path is**
- **The state of N particles is affected by a 3N dimensional field, which guides the motion of the particles**
  
  De Broglie called this the **pilot wave**; Bohm called it the **ψ-field**. This field has a piloting influence on the motion of the particles. The **quantum potential** is derived from the **ψ-field**.
  
  - **This 3N dimensional field satisfies the Schrödinger equation**
    
    Mathematically, the field corresponds to the **wavefunction** of conventional quantum mechanics, and evolves according to the **Schrödinger equation**. The positions of the particles do not affect the wave function.
  
  - **Each particle's momentum p is \( \nabla S(x, t) \)**
  
  - **The particles form a statistical ensemble, with probability density**
    
    \[ \rho(x, t) = |\psi(x, t)|^2 \]

**Reformulating the Schrödinger equation**

Some of Bohm's insights are based on a reformulation of the **Schrödinger equation**: instead of using the wavefunction \( \psi(x, t) \), he defines the wavefunction as

\[ \psi(x, t) = R(x, t)e^{iS(x,t)/\hbar} \]
and solves it for the (real) magnitude function $R(x, t)$ and (real) phase function $S(x, t)$.

The **Schrödinger equation** for one particle of mass $m$ is

$$i\hbar \frac{\partial \psi(x, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(x, t) + V(x)\psi(x, t),$$

where the **wavefunction** $\psi(x, t)$ is a complex function of the spatial coordinate $x$ and time $t$. It can then be split into two coupled equations by expressing it in terms of $R$ and $S$:

$$\frac{\partial R(x, t)}{\partial t} = -1 \frac{1}{2m} \left[ R(x, t) \nabla^2 S(x, t) + 2 \nabla R(x, t) \cdot \nabla S(x, t) \right]$$

$$\frac{\partial S(x, t)}{\partial t} = - \left[ V + \frac{1}{2m} (\nabla S(x, t))^2 - \frac{\hbar^2}{2m} \frac{\nabla^2 R(x, t)}{R(x, t)} \right].$$

The probability density $\rho(x, t)$ is a real function defined as the magnitude of the wave function:

$$\rho(x, t) = |\psi(x, t)|^2 = R^2(x, t)$$

Thus

$$\psi(x, t) = \sqrt{\rho(x, t)} e^{iS(x, t) / \hbar}.$$ 

Therefore we can substitute $\rho(x, t)$ for $R^2(x, t)$ and get:

$$- \frac{\partial \rho(x, t)}{\partial t} = \nabla \cdot \left( \rho(x, t) \frac{\nabla S(x, t)}{m} \right) \quad (1)$$

$$- \frac{\partial S(x, t)}{\partial t} = V(x) + Q(x, t) + \frac{1}{2m} (\nabla S(x, t))^2 \quad (2)$$

where

$$Q(x, t) = - \frac{\hbar^2}{2m} \frac{\nabla^2 R(x, t)}{R(x, t)} = - \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho(x, t)}}{\sqrt{\rho(x, t)}}$$

Bohm called the function $Q(x, t)$ the **quantum potential**.

We can use the same argument on the many-particle **Schrödinger equation**: 
where the $i$-th particle has mass $m_i$ and position coordinate $\mathbf{x}_i$ at time $t$. The wavefunction $\psi(x_1, x_2, \ldots, t)$ is a complex function of all the $\mathbf{x}_i$ and time $t$. $\nabla_i$ is the grad operator with respect to $\mathbf{x}_i$, i.e. of the $i$-th particle's position coordinate. As before the probability density $\rho(x_1, x_2, \ldots, t)$ is a real function defined by

$$\rho(x_1, x_2, \ldots, t) = |\psi(x_1, x_2, \ldots, t)|^2.$$ 

As before, we can define a real function $S(x_1, x_2, \ldots, t)$ to be the complex phase, so that we can define a similar relationship to the 1-particle example:

$$\psi(x_1, x_2, \ldots, t) = \sqrt{\rho(x_1, x_2, \ldots, t)} e^{i S(x_1, x_2, \ldots, t)/\hbar}.$$ 

We can use the same argument to express the Schrödinger equation in terms of $\rho(x_1, x_2, \ldots, t)$ and $S(x_1, x_2, \ldots, t)$:

$$-\frac{\partial \rho(x_1, x_2, \ldots, t)}{\partial t} = \sum_i \nabla_i \cdot \left( \rho(x_1, x_2, \ldots, t) \frac{\nabla_i S(x_1, x_2, \ldots, t)}{m_i} \right)$$

(3)

$$-\frac{\partial S(x_1, x_2, \ldots, t)}{\partial t} = V(x_1, x_2, \ldots) + Q(x_1, x_2, \ldots, t) + \sum_i \frac{1}{2m_i} (\nabla_i S(x_1, x_2, \ldots, t))^2$$

(4)

where

$$Q(x_1, x_2, \ldots, t) = -\sum_i \frac{\hbar^2}{2m_i} \frac{\nabla_i^2 R(x_1, x_2, \ldots, t)}{R(x_1, x_2, \ldots, t)} = -\sum_i \frac{\hbar^2}{2m_i} \left( \frac{\nabla_i^2 \rho(x_1, x_2, \ldots, t)}{2\rho(x_1, x_2, \ldots, t)} - \left( \frac{\nabla_i \rho(x_1, x_2, \ldots, t)}{2\rho(x_1, x_2, \ldots, t)} \right)^2 \right).$$

One-particle formalism

In his 1952 paper, Bohm starts from the reformulated Schrödinger equation. He points out that in equation (2):

$$-\frac{\partial S(x, t)}{\partial t} = V(x) + Q(x, t) + \frac{1}{2m} (\nabla S(x, t))^2,$$

if one represents the world of classical physics by setting $\hbar$ to zero (which results in $Q$ becoming zero) then $S(x, t)$ is the solution to the Hamilton-Jacobi equation. He
quotes a theorem that says that if an ensemble of particles (which follow the equations of motion) have trajectories that are normal to a surface of constant $S$, then they are normal to all surfaces of constant $S$, and that $\nabla S(x, t)/m$ is the velocity of any particle passing point $X$ at time $t$.

Therefore, we can express equation (1) as:

$$-\frac{\partial \rho(x, t)}{\partial t} = \nabla \cdot (\rho(x, t)v) .$$

This equation shows that it is consistent to express $\rho(x, t)$ as the probability density because $\rho(x, t)v$ is then the mean current of particles, and the equation expresses the conservation of probability.

Of course, $\tilde{\hbar}$ is non-zero. Bohm suggests that we still treat the particle velocity as $\nabla S(x, t)/m$. The movement of a particle is described by equation (2):

$$-\frac{\partial S(x, t)}{\partial t} = V(x) + Q(x, t) + \frac{1}{2m}(\nabla S(x, t))^2$$

where

$$Q(x, t) = -\frac{\hbar^2}{2m} \left( \frac{\nabla^2 \rho(x, t)}{2\rho(x, t)} - \left( \frac{\nabla \rho(x, t)}{2\rho(x, t)} \right)^2 \right) .$$

$V$ is the classical potential, which influences the particle's movement in the ways described by the classical laws of motion. $Q$ also has the form of a potential; it is known as the quantum potential. It influences particles in ways that are specific to quantum theory. Thus the particle is moving under the influence of a quantum potential $Q$ as well as the classical potential $V$.

**Many-particle formalism**

The momentum of Bohm's $i$-th particle's "hidden variable" is defined by

$$p_i = m_i v_i = \nabla_i S \quad (3)$$

and the particles' total energy as $E = -\partial S/\partial t$; equation (1) is the continuity equation for probability with

$$j_i = \rho v_i = \rho \frac{p_i}{m_i} = \rho \frac{\nabla_i S}{m_i} ,$$

and equation (4) is a statement that total energy is the sum of the potential energy, quantum potential and the kinetic energies.